Mechanics and Mechanical Engineering
Vol. 22, No. 4 (2018) 1223–1238

https://doi.org/10.2478/mme-2018-0094

Kinematic and Dynamic Analysis for a New MacPherson Strut Suspension System

S. Dehbari, J. Marzbanrad
Vehicle Dynamical Systems Research Laboratory, School of Automotive Engineering
Iran University of Science and Technology, Tehran, Iran
e-mail: dehbari@alumni.iust.ac.ir; marzban@iust.ac.ir

Received (21 November 2017)
Revised (11 March 2018)
Accepted (10 September 2018)

The present paper undertakes kinematic and dynamic analysis of front suspension system. The investigated model is a full-scale Macpherson which is a multibody system. Two degree of freedom model is considered here to illustrate the vertical displacement of sprung mass and unsprung mass with using displacement matrix. Ride and handling parameters including displacement of sprung and unsprung masses, camber/caster angle, and track changes are derived from the relationships. Moreover, geometrical model and equations are validated by Adams/Car software. The kinematic and dynamic results have been compared in both analytical and numerical outputs for verification. The proposed analytical model shows less than 5% differences with a complicated multibody model.

Keywords: suspension system, MacPherson, multibody system, ride, camber, caster.

1. Introduction

Suspension system is composed of a set of links connecting wheels to the car body. The main duties of the suspension system are isolated car body from road input and keeping wheels on appropriate position. Without suspension system, car body moves drastically and parts of the car will shortly fail due to the shocks imposed by road.

Suspension system effects on ride and vertical response. The primary functions of suspension system are providing vertical compliance and maintaining the wheels in the proper steer and camber attitudes to the road surface [1]. This latter function, which is related to car handling, depends on the type of suspension system its geometry.

The McPherson strut is a very popular mechanism for independent front suspension of small and mid-size cars. Fig. 1 illustrates a McPherson strut suspension. The McPherson strut, also called the Chapman strut, was invented by Earl McPherson in the 1940's. It was first introduced on the 1949 Ford Vedette, and also adopted in the 1951 Ford Consul, and then become one of the dominating suspension systems because it's compactness and has a low cost [2].

Of the weak points in this system, one can refer to its poor handling compared to other models such as double wishbone suspension system.

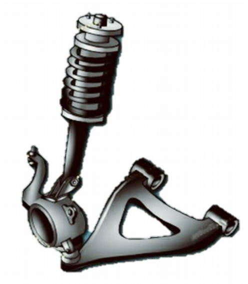

Figure 1 McPherson Suspension System

MacPherson suspension system composed of control arm, strut, knuckle, and connections. This suspension system is attached to the car frame via the control arm at the bottom of the system. This arm is connected to the knuckle via a ball joint. As can be seen on the Fig. 1, left rod of the knuckle is to the steering arm. Finally, the center of mechanism is connected to the wheel assembly. Upper part of the knuckle is where strut assembly is mounted. Strut is composed of spring and damper and bears the weight of the car.

So far numerous researches have been performed to analyse the performance of MacPherson suspension system with conventical two-degree of freedom (DOF). Sharma et al. used a two-DOF model to analyze the ride comfort with only spring, damper and tire stiffness [3]. Two-DOF model was also used to optimize of sliding mode control for a vehicle suspension system [4]. In research of Marzbanrad and Zahabi active control of a quarter-car vehicle model is investigated according to Two-DOF model [5]. Furthermore, this model has been used to analyze semi-active suspension systems. Chi et al. used a two-DOF model for design optimization of vehicle suspensions [6]. Finally, the conventional two-DOF model was used for experimental analysis [7]. Despite its good capabilities for investigating ride comfort, it cannot analyze many suspension system related parameters such as camber/caster angle of the car.

However, some researchers have considered two-dimension (2D) model of MacPherson suspension system, but the relationships of parameters have not been thoroughly discussed. In the papers of Fallah et al. and Hurel et al., 2D model of the suspension system and its components is investigated [8, 9]. These models calculate many of the suspension system's parameters, such as camber and king-pin angles, vertical wheel displacements, sprung mass, and track. 2D models have been also used to simulate semi-active suspension systems for ride control [10].

There are rare researches with three-dimensional suspension system model. Mantaras et al. used a 5-DOF 3D model [11]. In this article, suspension system with along steering system were simulated though, this model has complicated equations without calculation main parameters like track changes.

Moreover, in some researches on suspension system, just structure of one part of suspension is investigated. Mark et al. optimized arm control of a suspension system [12]. In his paper, dynamics and kinematics are ignored and applied forces on control arm and its response are considered.

Proper design of the suspension system plays an essential role in achieving optimal performance because ride and handling are compromising. In other words, one tends to be deteriorated as the other one is being improved. Therefore, in order to properly design a suspension system, one should consider its precise mathematical model. Since some of the behaviors of a suspension system, such as camber angle, depend on instantaneous geometry of the arms in the suspension system, to inquiring further details, researcher should use a model which includes all components of the system. Due to suspension system is a multi-body system and exhibits nonlinear behavior, the considered mathematical model shall be capable of correctly predicting the behavior of the suspension system. Multibody system simulation approach is a popular method of investigation of suspension system [13].

The used parameters in this research have been derived from Dacia Logan platform. The model is plotted according to exact dimensions and can analysis both kinematic parameters including camber and caster angles and track changes and also dynamic parameters like displacement imposed to car masses by the road.

The paper is arranged as follows. Section 1 presents the main idea discussed in this paper and introduces a literature review. In Section 2, first of all a new mathematical model of the Macpherson suspension is presented. Then the degree of freedom of the system is determined. After that, by using the displacement matrices, the kinematic relations of the system and different parameters like camber, caster angles and also track width changes are derived. In the next section, the Lagrange method is employed to analyze dynamic character. This is followed by the model verification in which the Adams Model and road input and comparative outputs of two models, i.e. the analytical and numerical models are represented. Conclusion and acknowledge are presented in Sections 5 and 6, respectively.

2. Two-Dimensional Model

2.1. *Geometrical Model*

The 2D geometrical model is plotted from the Catia model of the car suspension system (Fig. 2) provided to the authors by the platform design team.

For this purpose, first, the point O at which the control arm was connected to the car frame was taken as origin, and then coordinates of other points were calculated accordingly. At the point O, exist a revolute joint. The control arm is connected to the knuckle via a ball joint. Strut assembly (including spring and damper) is fixed to the knuckle. The steering rod is also connected to the knuckle via a ball joint, which is of course invisible in the 2D model.

Since the plotted model is very close to the reality, it allows us to predict the behavior of the suspension system precisely. Components of the drawn model (Fig. 3) are as follows:

1. Unsprung mass (M_s)

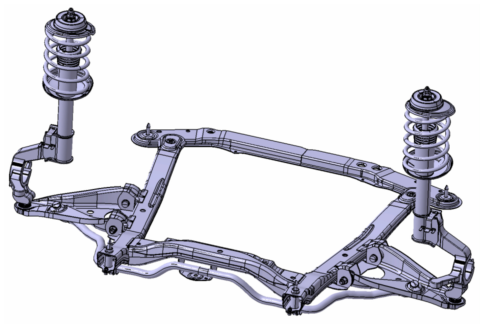

Figure 2 Catia model of Dacia Logan chassis

2. Control arm (OE)
3. Knuckle (CDE)
4. Strut (AB)
5. Wheels

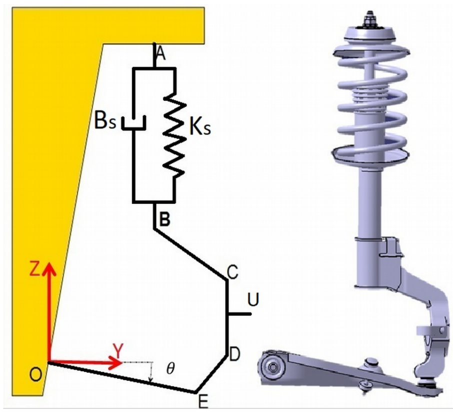

Figure 3 McPherson suspension system left: analytical model right: Catia model

2.2. *Determination of DOF*

In order to analyze a kinematic and dynamic of a multibody system, first, one should determine its degrees of freedom. For this purpose, Kutzbach method is employed [14]

$$F = b\,(n-1) - \sum_{i=1}^{j} (b - f_i) \; . \tag{1}$$

In this equation, F represents the DOF for closed system, b is the maximum DOF for a link in 2D space (here is 3), n is the number of links, f_i represents the DOF of the i-th joint, and j denotes the number of joints. MacPherson suspension system is composed of 5 members including 4 links (A, BE, OE, and Ms) and the ground. There are three revolute joints at O, E, and A, with a cylindrical joint for the unsprung mass, each of which reduces 2 degree of freedom; further, one of the joints is point-fixed and reduces the DOF by 3. Since the strut assembly is extendable, which itself presents 1 DOF, it adds the system DOF by 1. Totally by using Eq. (1) the model has 2 degree of freedom.

The two-DOF refer to the movement of the sprung mass and wheel along z-axis. Therefore, locations of all points can be written in terms of these two variables. The following assumptions were taken for this model.

1. Due to symmetry in a car, sprung mass will move only along z-axis.
2. All joints are ideal.
3. The values of spring, damper, and wheel stiffness behave linearly.
4. Masses of control arm and sequential arm are negligible.

2.3. *Kinematics*

Kinematic equations are derived by the displacement matrix [15]. It is one of the most useful representation method of rigid-body rotations, based on directional cosines [16]. Used in Fallah et al. [17], this method determines points coordinates in multi-DOF systems. If a multi-DOF system rotates by the angle θ in 2D space, the corresponding displacement matrix is calculated as follows:

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{2}$$

If the system does not rotate, but rather is solely shifted along axes of coordinates, the displacement matrix, d, is equal to:

$$d = \begin{bmatrix} d_{11} \\ d_{12} \end{bmatrix} \tag{3}$$

In case system rotates and shifts together, the corresponding displacement matrix will be a combination of the above-mentioned matrixes, i.e.:

$$H = \begin{bmatrix} R_{2\times 2} & d_{2\times 1} \\ f_{1\times 2} & s_{1\times 1} \end{bmatrix} \tag{4}$$

Vertical road input is applied to the wheel at the point U and vibrates the wheel vertically. These vibrations are transmitted from the wheel to the suspension system and knuckle. The knuckle is connected to the control arm at point E. The control arm rotates around the point O and converts the vertical movement to a horizontal one. This moves the spindle horizontally and creates camber angle. Vertical vibrations are further transmitted to the sprung mass via spindle.

The main points under consideration are the points B, D, and E. Each of these points have two components in the two-dimensional space of $Y - Z$. The input vibrations rotate the suspension system around x-axis by the angle Φ via the wheel and further generate two translational movements along y- and z-axes. The displacement matrix is written as follows

$$[D] = \begin{bmatrix} a_{11} & a_{12} & d_{11} \\ a_{21} & a_{22} & d_{22} \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

In this matrix, a is defined as below:

$$\begin{aligned} a_{11} &= a_{22} = \cos(\varphi) \\ a_{12} &= -a_{21} = \sin(\varphi) \end{aligned} \tag{6}$$

d_{11} and d_{12} in the displacement matrix represent the shifts along y- and z-axes respectively. In order to determine these two parameters, one can use the following method:

$$\begin{bmatrix} Y_C \\ Z_C \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & d_{11} \\ a_{21} & a_{22} & d_{22} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_{C0} \\ Z_{C0} \\ 1 \end{bmatrix} \tag{7}$$

As a result:

$$\begin{aligned} d_{11} &= Y_U - (a_{11}Y_{U0} + a_{12}Z_{U0}) \\ d_{12} &= Z_U - (a_{21}Y_{U0} + a_{22}Z_{U0}) \end{aligned} \tag{8}$$

where Z_U and Y_U are equal to:

$$Z_U = Z_{U0} + z_u \,, \tag{9}$$

$$Y_U = Y_{U0} + y_u \,. \tag{10}$$

Therefore, the displacement matrix can be calculated as follows:

$$[D] = \begin{bmatrix} a_{11} & a_{12} & Y_U - (a_{11}Y_{U0} + a_{12}Z_{U0}) \\ a_{21} & a_{22} & Z_U - (a_{21}Y_{U0} + a_{22}Z_{U0}) \\ 0 & 0 & 1 \end{bmatrix} \tag{11}$$

One can obtain the locations of points including B, D, E (see Fig. 3) as follows:

$$\begin{bmatrix} Y_B & Y_D & Y_E \\ Z_B & Z_D & Z_E \\ 1 & 1 & 1 \end{bmatrix} = [D] \times \begin{bmatrix} Y_{B0} & Y_{D0} & Y_{E0} \\ Z_{B0} & Z_{D0} & Z_{E0} \\ 1 & 1 & 1 \end{bmatrix} \tag{12}$$

Thus, the locations may be derived as:

$$\begin{aligned} Y_B &= a_{11}Y_{B0} + a_{12}Z_{B0} + Y_U - (a_{11}Y_{U0} + a_{12}Z_{U0}) \\ Z_B &= a_{12}Y_{B0} + a_{22}Z_{B0} + Z_U - (a_{21}Y_{U0} + a_{22}Z_{U0}) \\ Y_D &= a_{11}Y_{D0} + a_{12}Z_{D0} + Y_U - (a_{11}Y_{U0} + a_{12}Z_{U0}) \\ Z_D &= a_{21}Y_{D0} + a_{22}Z_{D0} + Z_U - (a_{21}Y_{U0} - a_{22}Z_{U0}) \\ Y_E &= a_{11}Y_{E0} + a_{12}Z_{E0} + Y_U - (a_{11}Y_{U0} + a_{12}Z_{U0}) \\ Z_E &= a_{21}Y_{E0} + a_{22}Z_{E0} + Z_U - (a_{21}Y_{U0} + a_{22}Z_{U0}) \end{aligned} \tag{13}$$

Since the angle $\varPhi$ is less than 6^o, the equations can be linearised with following assumptions:

$$\begin{aligned} a_{11} &= a_{22} = \cos(\varphi) \cong 1 \\ a_{12} &= -a_{21} = \sin(\varphi) \cong \varphi \end{aligned}$$

Accordingly, in Eq. (13) there will be 6 equations with 9 unknowns including (YB, ZB), (YC, ZC), (YD, ZD), (YE, ZE), and Φ.

2.4. *Camber*

Camber angle is the angle between the wheel plane and the vertical line. It is positive when the top of the wheel leans outward (Fig. 4). This definition of sign is only applicable for axles with two wheels. For instance, it is not applicable for motorcycles. Camber generates lateral forces and gives tire wear. Camber force (also called camber thrust) is the lateral force caused by the cambering of a wheel [18].

The camber angle may be made by either of two causes: 1) camber by roll, and 2) camber by bump. A change in the angle of the control arm develops some camber angle, enhancing the tire-ground contact. It prevents the car from rollover. As such, this angle plays an important role in handling of car.

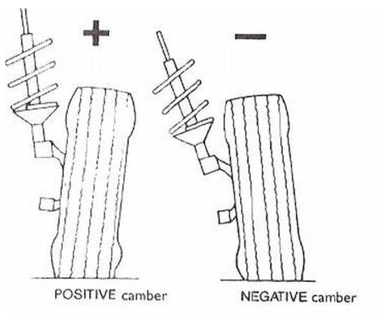

Figure 4 Camber angle [19]

In order to calculate the camber angle, it is assumed that sprung mass is constant and the wheels displaced vertically. Since the whole system rotates by Φ, the camber angle is calculated as follows:

$$\varphi = \frac{Y_B - Y_A}{Z_B - Z_A} \tag{14}$$

In this suspension system model, initial value of the camber angle is 0 and the link AB is normal to the ground. According to the assumptions, Y_A does not change. According to Eq. (13) the parameters Y_B and Z_B are functions of the angle φ, z_u, and y_u. As the system is a 2-DOF, all of the variables are functions of z_s and z_u. One can rewrite y_u in terms of these two variables. For this purpose, at first the coordinates of Y_E should be calculated.

$$y_u = Y_E - Y_{E0} + \phi\left(Z_{U0} - Z_{E0}\right) \tag{15}$$

By considering the link OE:

$$Y_E = \sqrt{\left(L_{OE}\right)^2 - Z_E^2} \tag{16}$$

Combining Eq. (13), (15), and (16) and substituting Z_E gives:

$$\phi = \frac{Y_{B0}+\phi(Z_{B0}-Z_{U0})+\sqrt{L_{OE}^2-(\phi(Y_{U0}-Y_{E0}+Z_{E0}+z_u))^2}-Y_{E0}+\phi(Z_{U0}-Z_{E0})-Y_A}{\phi(Y_{U0}-Y_{B0})+(Z_{B0}+z_u)-Z_{A0}-z_s} \tag{17}$$

In order to simplify the relations, the following parameters are assumed:

$$a = (Y_{U0} - Y_{B0})$$

$$b = (z_u - Z_{A0} - z_s + Z_{E0})$$
$$c = (Y_{B0} - Y_{E0} - Y_A)$$
$$d = (Y_{U0} - Y_{E0})$$
$$e = (Z_{E0} + z_u)$$
$$f = \sqrt{(-2cb + 2ed)^2 - 4\,(b^2 + d^2)\,(c^2 + e^2 - L_{FE}^2)}$$

Substituting these assumptions in Equation (17) gives:

$$a\,\varphi^2 + b\varphi - c = \sqrt{(L_{OE})^2 - (d\varphi + e)^2} \tag{18}$$

Due to small value of the angle, the second-order of φ can be neglected. By squaring the equation:

$$(b\varphi - c)^2 = L_{FE}^2 - (d\varphi + e)^2 \tag{19}$$
$$b^2\varphi^2 - 2bc\varphi + c^2 = L_{FE}^2 - d^2\varphi^2 - 2d\varphi - e^2 \tag{20}$$

After reordering:

$$\varphi^2\left(b^2 + d^2\right) + \varphi(-2cb + 2de) + \left(c^2 + e^2 + L_{FE}^2\right) \tag{21}$$

$$\varphi = \frac{(2de - 2cb) + \sqrt{(-2cb + 2ed)^2 - 4\,(b^2 + d^2)\,(c^2 + e^2 + L_{FE}^2)}}{2(b^2 + d^2)} \tag{22}$$

This equation gives the camber angle as a function of z_s and z_u.

2.5. ***Track Changes***

As stated earlier, when tire moves vertically respect to the body, track will change. On the real vehicle, the displacements of the tire contact patch relative to the road wheel would also result due to the effects of tire distortion [20].

$$y_u = Y_E - Y_{E0} + \varphi\,(Z_{U0} - Z_{E0}) \tag{23}$$

$$Y_E = \sqrt{L_{OE}^2 - Z_E^2} \tag{24}$$

$$Z_E = \varphi\,(Y_{U0} - Y_{E0}) + (Z_{E0} + z_u) \tag{25}$$

$$y_u = \sqrt{L_{OE}^2 - (\varphi\,(Y_{U0} - Y_{E0}) + (Z_{E0} + z_u))^2} - Y_{E0} + \varphi\,(Z_{U0} - Z_{E0}) \tag{26}$$

The angle θ can be also calculated as a function of camber angle.

$$\theta = \sin^{-1}\frac{Z_{E0} - Z_O}{L_{OE}} \tag{27}$$

$$\theta = \sin^{-1}\frac{\varphi\,(Y_{U0} - Y_{E0}) + (Z_{E0} + z_u) - z_s}{L_{OE}} \tag{28}$$

2.6. *Kingpin*

According to ISO 8855, the kingpin inclination is the angle σ which arises between the steering axis and a vertical to the road [21]

$$\sigma = \frac{Y_D - Y_A}{Z_D - Z_A} \tag{29}$$

2.7. *Caster*

In vehicles, bikes, and bicycles, caster angle is defined as the angle between the steer axis and the vertical line in the lateral plane $(X - Y)$. This angle is between an auxiliary line passing through the center of the wheel and the upper ball joint, in one side, and a horizontal line, on the other side. Looking the car from right, if the steering axis is in the right of the vertical line, the caster angle is positive, and it is negative if the steering axis is in the left of the vertical axis.

Caster variation is not generally desirable, but is used deliberately in some cases, for example to offset the effect of body pitch in braking. The caster angle and axis offset (the kingpin axis to wheel center separation in side view) give the caster trail, which, in conjunction with the tire pneumatic trail, is very important in giving the steering a suitable feel, and also has a significant effect on directional stability because of steering compliance [22].

Negative caster aids in centering the steering wheel after a turn and makes the front tires straighten quicker. Most street cars are made with $4 - 6$ deg negative caster. Negative caster tends to straighten the wheel when the vehicle is travelling forward, as a result is used to enhance straight-line stability

$$\varphi = \frac{X_B - X_A}{Z_B - Z_A} \tag{30}$$

2.8. *Dynamics*

In this section, dynamical analysis of the model is presented. When the car moves on a bumpy road, a force is applied by the road to the tires. This force is transmitted, via the arms of the suspension system, to the unsprung mass and passengers. There are numerous methods to calculate the acceleration applied to the passengers; in the present research, Lagrange equation is used for this purpose.

Lagrange's equation is a second-order partial differential equation that solutions are the functions for which a given functional is stationary [23].

Lagrange method has been applied to analyze many dynamic systems [24]. In this method, at first, kinetic force, T, potential force, V and damping force, D, should be calculated as follows:

$$T = \frac{1}{2}m_s {z'}_s^2 + \frac{1}{2}m_u {z'}_u^2 + \frac{1}{2}m_u {y'}_u^2 + \frac{1}{2}I_u {\varphi'}^2 + \frac{1}{2}I_{OE}{\theta'}^2 \tag{31}$$

$$V = \frac{1}{2}K_S\,\Delta L^2 + \frac{1}{2}K_T\,(z_u - z_r)^2 + \frac{1}{2}K_{TY}\,{Y_U}^2 \tag{32}$$

$$D = \frac{1}{2}B_S \Delta\dot{L}^2 \tag{33}$$

In Eq. (31), M_s and M_u are sprung and unsprung masses, respectively, z'_s and z'_u represent vertical velocities of the sprung and unsprung masses. Y'_u is horizontal velocity of the unsprung mass, and I_u and I_{OE} are moments of inertia of strut and control arm around x-axis. In Eq. (32), K_s refers to the stiffness of the primary spring, ΔL is the length change of the strut, K_T is the stiffness of the car tire along vertical direction, and K_{TY} represents the spring stiffness in horizontal direction, with z_r denoting road input. Damping coefficient of the damper in Eq. (33) is shown by B_s. Lagrangian is defined as the difference of kinetic and potential energies.

$$L = T - V \tag{34}$$

Then, one should differentiate respect to time and also respect to independent variables z_s and z_u as follows:

$$\begin{aligned} L = &\frac{1}{2}m_s {z'}_s^2 + \frac{1}{2}m_u {z'}_u^2 + \frac{1}{2}m_u {y'}_u^2 + \frac{1}{2}I_u {\varphi'}^2 + \frac{1}{2}I_{OE}{\theta'}^2 \\ &-\frac{1}{2}K_S \Delta L^2 - \frac{1}{2}K_T (z_u - z_r)^2 - \frac{1}{2}K_{TY} y_u^2 \end{aligned} \tag{35}$$

Then, one should differentiate respect to time and also respect to independent variables z_s and z_u as follows:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{z'}_s}\right] - \left[\frac{\partial L}{\partial z_s}\right] + \left[\frac{\partial D}{\partial \dot{z}_s}\right] = 0 \tag{36}$$

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{z'}_u}\right] - \left[\frac{\partial L}{\partial z_u}\right] + \left[\frac{\partial D}{\partial \dot{z}_u}\right] = 0 \tag{37}$$

by expanding Eqs. (36) and (37):

$$\begin{aligned} &m_s z''_s + m_u \left(y''_u \tfrac{\partial {y'}_u}{\partial {z'}_s} + y'_u \tfrac{d}{dt}\left(\tfrac{\partial {y'}_u}{\partial {z'}_s}\right)\right) + I_u \left(\varphi'' \tfrac{\partial \varphi'}{\partial {z'}_s} + \varphi' \tfrac{d}{dt}\left(\tfrac{\partial \varphi'}{\partial {z'}_s}\right)\right) + \\ &I_{OE}\left(\theta'' \tfrac{\partial \theta'}{\partial z'_s} + \theta' \tfrac{d}{dt}\left(\tfrac{\partial \theta'}{\partial z'_s}\right)\right) - m_u y'_u - I_u \varphi' \tfrac{\partial \varphi'}{\partial z_s} - I_{OE}\theta' \tfrac{\partial \theta'}{\partial z_s} \\ &+K_S \Delta L \tfrac{\partial \Delta L}{\partial z_s} + K_{TY} y_u \tfrac{\partial y_u}{\partial z_s} = 0 \end{aligned}$$

$$\begin{aligned} &m_u z''_u + m_u \left(y''_u \tfrac{\partial y'_u}{\partial z'_u} + y'_u \tfrac{d}{dt}\left(\tfrac{\partial y'_u}{\partial z'_u}\right)\right) + I_u \left(\varphi'' \tfrac{\partial \varphi'}{\partial z'_u} + \varphi' \tfrac{d}{dt}\left(\tfrac{\partial \varphi'}{\partial z'_u}\right)\right) + \\ &I_{OE}\left(\theta'' \tfrac{\partial \theta'}{\partial z'_u} + \theta' \tfrac{d}{dt}\left(\tfrac{\partial \theta'}{\partial z'_u}\right)\right) - m_u y'_u \tfrac{\partial y'_u}{\partial z_u} - I_u \varphi' \tfrac{\partial \varphi'}{\partial z_u} - I_{OE}\theta' \tfrac{\partial \theta'}{\partial z_u} + \\ &K_T(z_u - z_r) + K_S \Delta L \tfrac{\partial \Delta L}{\partial z_u} + K_{TY} y_u \tfrac{\partial y_u}{\partial z_u} = 0 \end{aligned}$$

Due to complicated relations, just some typical resulted equations have been presented here. Strut length, including spring and damper, shall be calculated in the Lagrange equations.

$$\Delta L = L - L_0 \tag{38}$$

$$\Delta L = \sqrt{(Z_A - Z_B)^2 + (Y_A - Y_B)^2} - \sqrt{(Z_{A0} - Z_{B0})^2 + (Y_{A0} - Y_{B0})^2} \tag{39}$$

and its derivative is given by:

$$\Delta \dot{L} = 2\left(Z_A - Z_B\right)\left(\dot{z}_s - \dot{z}_u\right) + 2\left(Y_A - Y_B\right)\left(-\dot{\varphi} Z_{B0} - y_u + \dot{\varphi} Z_{U0}\right) \tag{40}$$

Derivative of the camber angle respect to time:

$$\begin{aligned} \phi = \Big\{ \; & 2(b^2+d^2)\left[2de' - 2cb' + \right. \\ & \left[(-2cb+2ed)(-2cb'+2de') - 4(2bb'(c^2+e^2-L_{FE}^2) - 4(b^2+d^2)(2ee')\right]/f\Big] \\ & -2bb'(2de-2cb+f) \; \Big\} : \Big\{ \; 4\left(b^2+d^2\right)^2 \; \Big\} \end{aligned} \tag{41}$$

The second derivative of the horizontal displacement of the unsprung mass with respect to time:

$$\begin{aligned} y_u'' = & -\frac{\left[\phi''(\phi(Y_{U0}-Y_{E0})+(Z_{E0}+z_u))+\dot{\phi}'^2(Y_{U0}-Y_{E0})\right]\sqrt{L_{OE}^2-(\phi(Y_{U0}-Y_{E0})+(Z_{E0}+z_u))^2}}{2\left[L_{OE}^2-\phi^2(Y_{U0}-Y_{E0}+Z_{E0}+z_u)^2\right]} \\ & +\phi''(Z_{U0}-Z_{E0}) \end{aligned}$$

After solving above equations and differentiate of different parameters respect to time, z_s and z_u, one can obtain the acceleration applied to the passengers.

The above equations were coded in MATLAB. This programing software has useful functions for solving dynamic and kinematic equations. Therefore, MATLAB and Simulink are frequently applied to analyze this type of systems [25]. In order to solve these differential equations, we used *ODE45* solver. Computation time was set to 10 s and the solution interval was divided into 500 steps.

3. Model Verification

3.1. *Adams Model*

For verifying the prepared analytical model, a suspension system with the same specifications was modeled in Adams/Car software, designed to analyse different parts of a car (e.g. suspension system), this software has been applied in numerous researches [26]. The system has been also modelled in Adams to simulate the same status for verification.

As it was explained in Sec. 1, dimensions of the suspension system were acquired from the corresponding Catia file. Coordinates of the points are presented in Tab. 1. Table 2 declares the value of non-geometrical suspension parameters.

Table 1 Coordination of the hard points

Hard points	X [mm]	Y [mm]	Z [mm]
O	0	0	0
A	35	200	650
B	5	200	225
C	-15	336	176
D	-35	336	16
E	-31	322	-40
U	-35	400	110

Table 2 Non-geometrical parameters value

Parameter	Value	parameter	value
M_s [kg]	280	M_u [kg]	50
K_s [N/m]	26185	B_s [Ns/m]	2166
K_T [N/m]	200000	K_{TY} [N/m]	180000
I_{OE} [kg m^2]	1	I_u	10

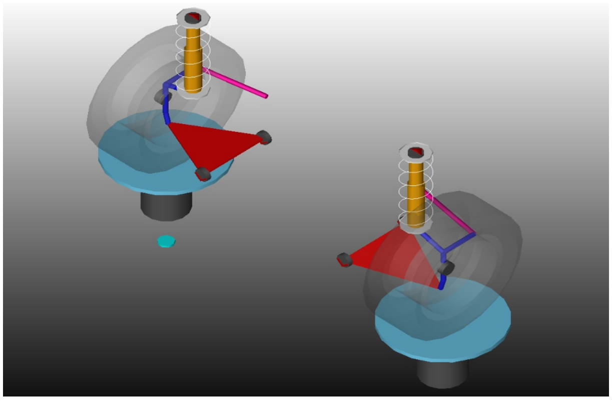

Figure 5 MacPherson Suspension system in Adams/Car

3.2. *Kinematic Analysis*

In order to analyse kinematic parameters of the suspension system, the wheel center was displaced by −80 to +80 mm. In this test, sprung mass of the car was assumed to be constant. The results are presented in Figs. 6 to 8.

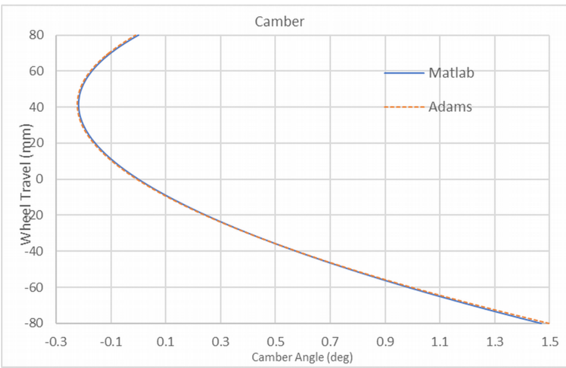

Figure 6 Camber angle vs. wheel travel

As can be seen, the results of the 2D model (coded in MATLAB) and 3D model (modeled in Adams) are so close to one another. The existing small deviations might be attributed to linearization which was explained in Sec. 2.

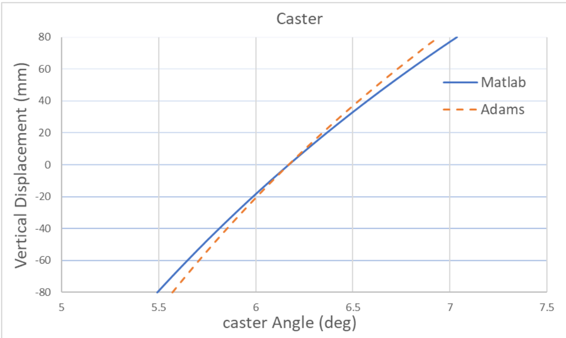

Figure 7 Caster angle vs. wheel travel

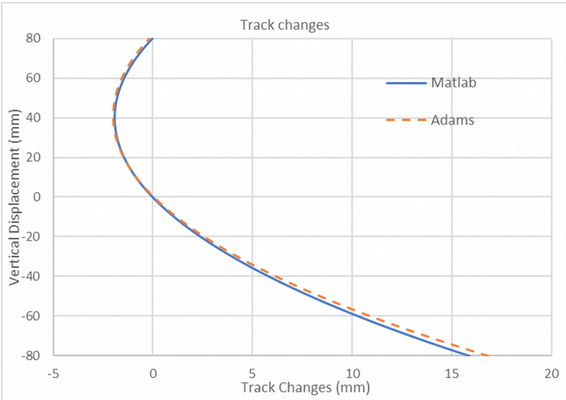

Figure 8 Track changes vs. wheel travel

3.3. *Dynamic Analysis*

To dynamic analyse, a 5cm step road input was applied to the both models (Fig. 9). The equation used to produce this plot in Adams/car is as follows:

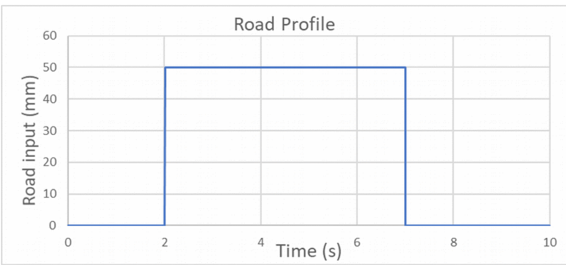

Figure 9 Road profile

$$zr = hav\sin(time, 2, 0, 2.01, 50) + hav\sin(time, 7, 0, 7.01, -50) \tag{42}$$

Simulation result is shown in Fig. 10.

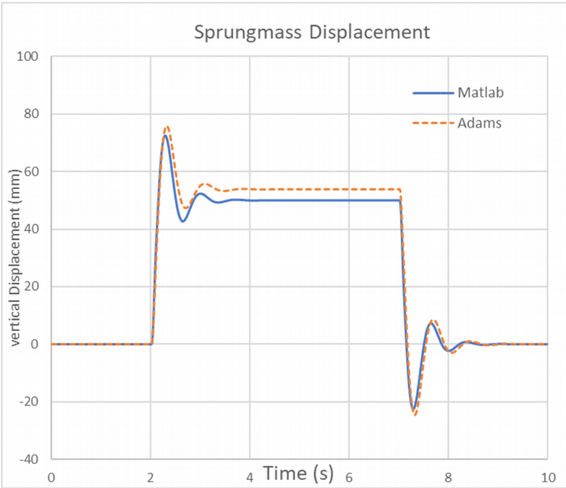

Figure 10 Sprung mass vertical displacement respect to time

4. Conclusions

McPherson suspension system is a frequently applied suspension system for small and mid-sized cars. Since this system exhibits non-linear behaviours, one should use appropriate models to analyse its ride and handling. Most of the existing literature on suspension systems have simulated the strut and tire only. Even though these models have been able to well analyse the ride, but they failed to consider the car handling due to not accounting for the geometry of the suspension system. In this paper, MacPherson suspension system of Dacia Logan was two-dimensionally modelled in full-size. This 2D model not only calculated the vertical acceleration applied to the car body, but also was capable of analysing many suspension system parameters such as caster angle and track which are related to the car handling. In order to validate the model, it was modelled in Adams/Car software. Two different tests were conducted on both models and the results were compared to one another. In the first test, in order to analyse kinematic parameters, center of the car tire was vertically displaced by -80 to $+80$ mm. In the second test, a 5 cm-step road input was applied to the tire. The differences between the outputs if analytical and 3D model in both tests was less than 5%. This model can be used to undertake frequency simulation, ride comfort and car safety analysis.

References

[1] **Gillspie, T.D.**: *Dynamics. Fundamentals of Vehicle*, Warrendale: Society of Automotive Engineers, Inc., **1992**.

[2] **Jazar, R.N.**: *Vehicle Dynamics. Theory and Applications*, New York: Springer, **2008**.

[3] **Sharma, P., Saluja, N., Saini, D. and Saini, P.**: Analysis of Automotive Passive Suspension System with Matlab Program, *International Journal of Advancements in Technology*, **4**(2), 115–119, **2013**.

[4] **Sharifi, M., Shahriari, B., Bagheri, A.**: Optimization of Sliding Mode Control for a Vehicle Suspension System via Multi-objective Genetic Algorithm with Uncertainty, *Journal of Basic and Applied Scientific Research*, **2012**.

[5] **Marzbanrad, J., Zahabi, N.**: H∞ Active Control of a Vehicle Suspension System Excited by Harmonic and Random Roads, *Mechanics and Mechanical Engineering*, **21**(1), 171–180, **2017**.

[6] **Chi, Z., He, Y. and Naterer, G.F.**: Design Optimization of Vehicle Suspensions with a Quarter-vehicle Model, *Transactions of the Canadian Society for Mechanical Engineering*, **32**(2), 297–312, **2008**.

[7] **Patil, S.A., Joshi, S.G.**: Experimental analysis of 2 DOF quarter-car passive and hydraulic active suspension systems for ride comfort, *Systems Science & Control Engineering*, **2**(1), 621–631, **2014**.

[8] **Fallah, M.S., Bhat, R., and Xie, W.F.**: New Nonlinear Model of Macpherson Suspension System for Ride Control Applications, in: *American Control Conference*, Seattle, **2008**.

[9] **Hurel, J., Mandow A., García-Cerezo A.**: Nonlinear Two-Dimensional Modeling of a McPherson Suspension for Kinematics and Dynamics Simulation, in: *International Workshop on Advanced Motion Control*, Sarajevo, Bosnia and Herzegovina, **2012**.

[10] **Fallah, M.S., Bhat, R. and Xie, W.F.**: New model and simulation of MacPherson suspension system for ride control application, *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, **47**(2), 195–200, **2009**.

[11] **Mantaras, D.A., Luque, P., Vera, C.**: Development and validation of a three-dimensional kinematic model for the McPherson steering and suspension mechanisms, *Mechanism and Machine Theory*, **39**, 603–619, **2003**.

[12] **Richard, M.J., Bouazara M., Khadir, L., Cai, G.Q.**: Structural Optimization Algorithm for Vehicle Suspensions, *Transactions of the Canadian Society for Mechanical Engineering*, **35**(1), **2011**.

[13] **Fichera, G. and Lacagnina, M.**: Modelling of Torsion Beam Rear Suspension by Using Multibody Method, *Multibody System Dynamics*, **12**, 303–316, **2004**.

[14] **Dicker J.J., Gordon, J.J., Pennock, R., Shigley, J.E.**: *Theory of Machines and Mechanisms*, New York, Oxford University Press, **2003**.

[15] **Spong, M.W., and Vidyasagar, M.**: *Robot Dynamics and Control*, New York: John Wiley & Sons, **1989**.

[16] **Jazar, R.N.**: *Advanced Dynamics*, New Jersey: John Wiley & Sons, **2011**.

[17] **Fallah, M.S., Mahzoon, M. and Eghtesad, M.**: Kinematical and Dynamical Analysis of Macpherson Suspension using Displacement Matrix Method, *Iranian Journal of Science & Technology, Transaction B, Engineering*, **32**(84), 325–339, **2008**.

[18] **Jacobson, B.**: *Vehicle Dynamics*, Göteborg: Chalmers, **2014**.

[19] **Esfahani, M.I.M., Mosayebi, M., Pourshams, M. and Keshavarzi, A.**: Optimization of Double Wishbone Suspension System with Variable Camber Angle by Hydraulic Mechanism, *International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing*, **4**(10), 60–67, **2010**.

[20] **Blundell, M. and Harty, D.**: *The Multibody Systems Approach to Vehicle Dynamics*, New York: Elsevier, **2004**.

[21] **Reimpell, J., Stoll, H. and Betzler, J.W.**: The Automotive Chassis: Engineering Principles, Warrendale: SAE international, **2002**.

[22] **Dixon, J.C.**: *Suspension Geometry and Computation*, Sussex: John Wiley and Sons, **2009**.

[23] **Pars, L.**.: *An Introduction to the Calculus of Variations*, London: Courier Dover Publications, **(1987)**.

[24] **Hurel, J., Mandow, A. and García-Cerezo, A.**: "Kinematic and dynamic analysis of the McPherson suspension with a planar quarter-car model, *Vehicle System Dynamics: International Journal of Vehicle Mechanics and Mobility*, **51**(9), 1422–1437, **2014**.

[25] **Khajavi, M.N., Notghi, B., and Paygane, G.**: A Multi Objective Optimization Approach to Optimize Vehicle Ride and Handling Characteristics, *World Academy of Science, Engineering and Technology*, **38**, 580–584, **2010**.

[26] **Eskandari, A., Mirzadeh, O. and Azadi, S.**: Optimization of a McPherson Suspension System Using the Design of Experiments Method, in: *SAE Automotive Dynamics, Stability & Controls Conference and Exhibition*, Novi, **2006**.